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Locating charging points for an electric fleet

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ABSTRACT: Centrale OO *is a pioneering project aiming to deploy in Paris a fleet of 100 % electric taxis. The significant initial investment and the current restricted vehicle autonomy give a high relevancy to the charging terminal location task. This task can be seen as a variant of the well-known capacitated covering location problem (CCLP). A mixed integer programming model suiting the features of the project is proposed. Its originality lies in the way a punctual demand is built to satisfy the usual requirements of such models, whereas the true demand in the project is attached to moving vehicles. The efficiency of the proposed model has been compared by simulation with other more classical approaches on different sets of randomly generated instances.*

KEYWORDS: *electric vehicles, charging terminal location, covering location problem, linear programming, simulation.*

1 INTRODUCTION

1.1 Context

Centrale OO[1] is a pioneering project aiming to deploy in Paris a fleet of 100 % electric taxis. The company in charge of the management of the fleet is the *Société du Taxi Electrique Parisien (STEP)*. The deploiement of such fleets finds is main motivation in sustainable issues: electric vehicles release almost no air pollutants at the place where they are operated and have less noise pollution than internal combustion engine vehicles. However, the main drawback of an electric vehicle is its weak autonomy – 80 km in the case of the *Centrale OO* project. The constraints of the management, as expressed by the *STEP*, are

- A taxi must never break down
- An opportunistic demand inside Paris and its suburbs must always be satisfied (legal environment of Paris)
- The number of booking demands accepted has to be maximized

The charging problem of the taxis must therefore be carefully adressed. At a strategic level, one aspect of this problem consists in determining the best location for the charging terminals. At a tactical level, a good assignement of the trips to the taxis is crucial. In taxi fleet management, two kind of requests can be differentiated: *booking requests* and *opportunistic requests*. The first ones can be immediate or in advance of travel and have to be processed by the taxi dispatching system which assigns the request to a taxi. The opportunistic requests correspond with the traditional taxi services picking up passengers at cab-ranks or from the side of the road. Of course, this kind of requests are not processed by the dispatching system.

This paper deals with the location issue. The significant initial investment (the cost of an electrical charching terminal is about 10.000 euros) and the restricted vehicle autonomy give a high relevancy to the charging terminal location task. Indeed, a wrong placement may in effect lead to a poor fleet management with vehicles having difficulties to charge the batteries due to charging terminals saturation or even with vehicles constantly running out of charge to keep operating. Our purpose is to propose a practical way for computing the "best" locations.

1.2 Model

A complete directed graph $G = (V, A)$ models the network. The vertices are points in the city at which trips start and finish. They can moreover be used to locate charge terminals. The arcs model the possible trips. The duration of a trip is a random variable T_a

[1] See the website http://taxioo.com/index.html for an artistic view.

of expectation τ_a.

The demand for each possible trip $a \in A$ is assumed to follow a Poisson process of rate λ_a. Actually, these demands are split between *booking demands* and *opportunistic demands*, see Section 5 for a more accurate description.

There are n taxis. A taxi consumes γ Wh by unit of time when it is moving. It stores ρ Wh by unit of time when it is charging.

The number of charge terminals is denoted by r. Several terminals can be located at the same vertex.

1.3 Main results

Finding the right locations for the charging terminals can be seen as a variant of the well-known capacitated covering location problem (CCLP). We modelize it as a mixed integer program. Its relevance is proved through simulations and comparisons with other location strategies.

1.4 Plan

The paper is organized as follows. In Section 2, the facility location problem and more particularly the covering location problem are briefly introduced. An upper bound for the maximal number of customers accepted by the system and different strategies for demand estimation are detailled in Section 3. In Section 4, a mixed integer linear programming for electric charging terminal positionning is proposed. The principles of the taxi behaviour simulator are briefly described in Section 5. Finally, Section 6 is dedicated to computational experiments.

2 LITERATURE REVIEW ON COVERING LOCATION PROBLEMS

The location problem was originally defined by A. Webber when he considered how to position a single warehouse minimizing the total distance between the warehouse and a set of customers (Webber, 1929). In 1964, Hakimi (Hakimi, 1964) defines the *P-median problem*, the problem consists in determining the best location for a set of limited facilities in order to minimize the sum of the weighted distances between the clients and the facilities serving these clients.

The problem increases its relevance during the last decades, high costs related to property acquisition and facility construction make facility location projects a critical aspect of strategic planning for a wide range of private and public firms. Indeed, the fact that facility location projects are long-term investments leads the researchers to focus on dynamic and stochastic location problems (see (Owen and Daskin, 1998) for a review of this extension of the problem). Another important variant of the problem is the *Capacitated Facility Location Problem (CFLP)* where facilities have a constraining upper limit on the amount of demand they can satisfy. An extension of the CFLP closely to our problem is the *Capacitated Facility Location Problem with Multiple facilities in the same site (CFLPM)*. In charging terminal location the positions of the terminals are not the only decision variables, the number of terminals at each position have to be fixed too.

However, in some real-world applications selecting the best location for distance minimization is not the best suitable choice. For example, in electric vehicle charging terminal location, like in other critical applications such as ambulance and fire terminal location, the interest is to guarantee that the different geographic zones are covered by a facility (closer than a previously fixed covering distance). This class of problems are known as *Covering Location Problems* (see (White and Case, 1974), (Schilling, Vaidyanathan and Barkhi, 1993) and more recently (Vijeyamurthy and Panneerselvam, 2010) for a complete review of covering problems). In that context, the covering issue can be sometimes modelized as a problem constraint. However, if the covering distance is fixed to a small value the problem might become unfeasible. The *Maximal Covering Location Problem (MCLP)* (Church and ReVelle, 1974) locates the facilities in order to maximize the number of covered customers (customers with a distance to the nearest facility smaller than an initial fixed distance). An extension of the problem very interesting for critical applications is the maximal covering with mandatory closeness problem which imposes a maximal distance (less stringent than the covering distance) between the geographical zones and the nearest facility (Church and ReVelle, 1974). These covering models implicitly assume that if a geographical zone is covered by a facility then the facility will be always available to serve the demand. However, in some applications, when facilities have a fixed capacity, being covered is not sufficient to guarantee the demand satisfaction. We find in the literature some models attempting to overcome this issue by maximizing the number of geographical zones covered by multiple facilities (Daskin and Stern, 1981; Hogan and ReVelle, 1986; Gendreau, Laporte and Semet, 1997).

We present in Section 4 the linear programming models proposed to solve the electric vehicle charging location problem.

3 MACROSCOPIC RELATIONS

3.1 General relations

Let use denote by $\tilde{\lambda}_a$ the average number of demands for a trip a that are accepted by unit of time. We have $\tilde{\lambda}_a \leq \lambda_a$.

Let $\tilde{\lambda} = \sum_{a \in A} \tilde{\lambda}_a$ be the average number of trips accepted by unit of time and let $\tau = \frac{1}{\tilde{\lambda}} \sum_{a \in A} \tilde{\lambda_a} \tau_a$ be the average duration of an accepted trip.

The energy consumption of the system by unit of time is $\gamma\tilde{\lambda}\tau$. The maximal rate of supply in energy is ρr. Therefore, we have the following inequality

$$\gamma\tilde{\lambda}\tau \leq \rho r \tag{1}$$

Another inequality can be derived, by considerations on the time needed to realize the different tasks. Let us consider a taxi over a time window of sufficiently large duration T. Denote by x the time during which it stores energy at a charge terminal. Over the time window, it spends in average $\frac{T\tilde{\lambda}\tau}{n}$ unit of time with a customer on bord. Therefore, we have

$$\frac{T\tilde{\lambda}\tau}{n} + x \leq T$$

During this duration x, it stores a quantity of energy that must cover in average the consumption over the time window. Hence

$$\frac{\gamma T\tilde{\lambda}\tau}{n} \leq \rho x$$

Combining these two inequalities leads to

$$(\gamma + \rho)\tilde{\lambda}\tau \leq n \tag{2}$$

Equations (1) and (2) can be summarized in the following inequality.

$$\tilde{\lambda} \leq \min\left(\frac{n}{(\gamma+\rho)\tau}, \frac{\rho r}{\gamma\tau}\right) \tag{3}$$

Knowing the number of taxis, their efficiency (encoded by γ), the numer of charging terminals, their efficiency (encoded by ρ), we have an upper bound on the number of trips that can be accepted by unit of time.

3.2 Estimating a pointwise demand in energy

All the covering location models take in input a pointwise demand. A way to build such a demand d_i attached to a vertex i consists in computing the energy needed by unit of time for the trips starting at i: it is precisely $\gamma \sum_{a \in \delta^+(i)} \lambda_a \tau_a$. Dividing this quantity by ρ provides the number of recharge terminals ensuring this supply. It suggests to define

$$d_i^{out} = \frac{\gamma}{\rho} \sum_{a \in \delta^+(i)} \lambda_a \tau_a$$

In the last equation, the pointwise demand is calculated considering the trips starting at vertex i. This model presumes that taxis charging tasks take place mostly at the origin of the trips. However, other realistic strategies can be also envisaged. One of these strategies is to consider that taxis charge the batteries at the end of the trips. A pointwise demand considering the energy consumed by unit of time for the trips arriving at i is also proposed. We can then define

$$d_i^{in} = \frac{\gamma}{\rho} \sum_{a \in \delta^-(i)} \lambda_a \tau_a$$

Finally, a third strategy is proposed considering a pointwise demand as a linear combination of d_i^{out} and d_i^{in}:

$$d_i^{mix} = \alpha d_i^{out} + (1-\alpha) d_i^{in}$$

4 ELECTRIC VEHICLES CHARGING TERMINAL LOCATION

The EV charging terminal location problem consists in determining the best locations of the charging terminals. The linear programming model has to take into account two important aspects. First, the charging terminals have to be conveniently spread over the geographical area in order to avoid remote geographical zones which difficult taxi operability and fleet management. The second aspect is that the model has to determine the number of charging points facilitating the charging process of the taxis by minimizing the risks of terminals saturation. For these purposes, we propose two models, one called the *P-median model*, the other the *Demand-based model*.

V is the set of geographical points of the problem and $J \subseteq V$ is the set of potential locations where the charging terminals can be located. The number of terminals is limited to r.

4.1 P-median model

Following Hakimi (Hakimi, 1964), we define x_j to be the decision variables indicating if a facility is located to the point j and y_{ij} to be the variables indicating that the geographical point i is assigned to the facility located in j. The linear program minimizing the sum of the distances between clients and facilities can be written as follows.

$$\min \sum_{i\in V}\sum_{j\in J} dist_{ij}y_{ij} \tag{4}$$

s.t.

$$\sum_{j\in J} y_{ij} = 1 \quad \text{for all } i\in V \tag{5}$$

$$y_{ij} \le x_j \quad \text{for all } i\in V,\ j\in J \tag{6}$$

$$\sum_{j\in J} x_j \le r \tag{7}$$

$$x_j \in\{0,1\} \quad \text{for all } j\in J \tag{8}$$

$$y_{ij} \in\{0,1\} \quad \text{for all } i\in V,\ j\in J \tag{9}$$

4.2 Demand-based model

Another approach consists in defining a model with two distances β_{far} and β_{close} as proposed by Church and ReVelle (Church and ReVelle, 1974). The idea is then to spread the terminals by fixing a maximal distance (β_{far}) between the different geographical zones and the nearest charging terminal and, at the same time, trying to maximize the demand that will be covered by a nearby charging terminal (β_{close}).

We can then define J_i^{far} (resp. J_i^{close}) as the subset of points in J at distance less than β_{far} (resp. β_{close}) from $i \in V$. Conversely, V_j^{close} is the set of points at distance less than β_{close} from the point $j \in J$.

Let x_j be the decision variable indicating the number of terminals located at point $j \in J$ and y_{ij} to be the fraction of the demand d_i for $i \in V$ covered by a charging terminal located in j at distance less than β_{close} from i.

The linear programming model proposed to solve the problem called *Demand-based model* is the following.

$$\max \sum_{j\in J}\sum_{i\in V_j^{close}} d_i y_{ij} \tag{10}$$

s.t.

$$\sum_{j\in J_i^{far}} x_j \ge 1 \quad \text{for all } i\in V \tag{11}$$

$$\sum_{j\in J_i^{close}} y_{ij} \le 1 \quad \text{for all } i\in V \tag{12}$$

$$\sum_{i\in V_j^{close}} d_i y_{ij} \le x_j \quad \text{for all } j\in J \tag{13}$$

$$\sum_{j\in J} x_j \le r \tag{14}$$

$$x_j \in \mathbb{Z}_+ \quad \text{for all } j\in J \tag{15}$$

$$y_{ij} \in \mathbb{R}_+ \quad \text{for all } i\in V,\ j\in J_i^{close} \tag{16}$$

The objective function (Eq. 10) consists in maximizing the pointwise demand covered by a charging terminal considering the distance β_{close}. Eq. 11 imposes that a geographical zone $i \in V$ must be covered at least for one charging terminal considering the distance β_{far}. Here the mandatory closeness is only required for the geographical zones closer than β_{far} from a potential charging terminal location in order to find a solution even if this constraint is violated for some geographical zones. We stress that an adequately β_{far} make possible to spread the charging terminals over the geographical area. Eq. 12 specifies that for each geographical zone $i \in V$ the sum of the fractions of demand covered by a charging terminal considering the distance β_{close} has to be less or equal to the unit. The idea here is that the demand of each geographical point can be satisfied by different charging terminals and our interest is to maximize the potential energy required being supplied by a terminal closer than β_{close}. Eq. 13 are the constraints linking the variables x_j with the variables y_{ij}. For a given potential charging terminal location J_j, this last set of variables can only be positive if a charging terminal is finally located to J_j, that means $x_j > 0$. Besides, thanks to the definition of the pointwise demand d_i, Eq. 13 also imposes that the demand allocated to a charging terminal cannot exceed its capacity. The goal of this constraint is to assign a larger number of terminals on the geographical points with a great demand decreasing that way the risk of saturation for the charging terminals. Finally, Eq. 14 limits the number of terminals of the problem.

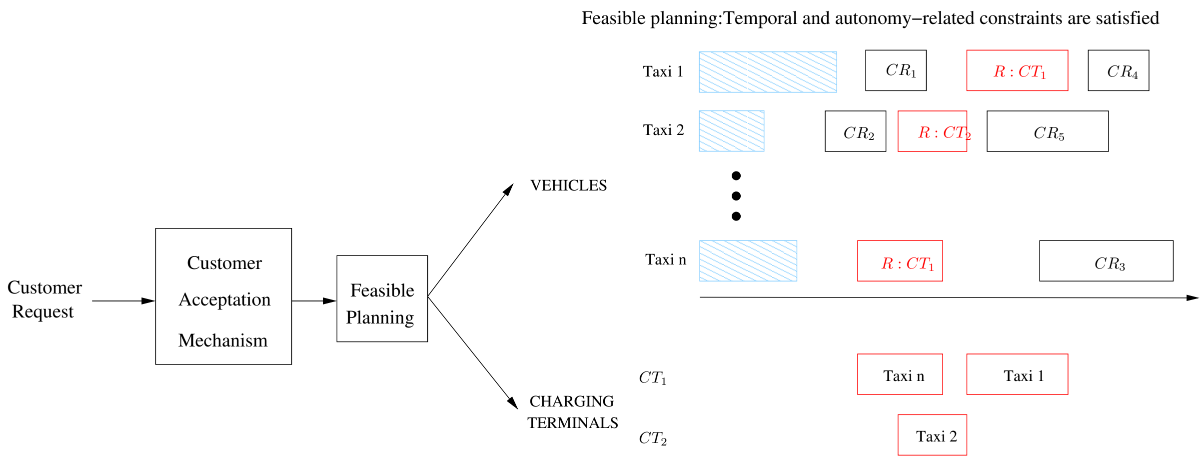

Figure 1: Customer acceptation mechanism for the electric vehicle management system architecture

5 TAXI SIMULATOR

5.1 Simulation

The taxi behavior simulator consists in a discrete-events simulator programmed in C++. It simulates the model described in Section 1.2. Each possible trip between the points of the city is characterized by two parameters, λ_{book} and λ_{opp}, representing the rates of the Poisson process for booked and opportunistic demand, respectively. For sake of simplicity, the duration and the distance of each trip are considered constant over the time. The inputs of the simulator are a description of the system state and the simulation parameters. The system is originally defined by:

- A set of points defined by their coordinates representing the geographical points of the city.
- The location of the charging terminals.
- The fleet of taxis. Each taxi is defined by the following parameters:
  - AUT: it is the current autonomy of the vehicle.
  - MAX_AUT: it is the autonomy of the vehicle when it is fully charged.
  - POS: it is the initial position of the vehicle.

Opportunistic and booking demands are treated differently. The booking demands are managed by the taxi dispatching system deciding whether a demand is accepted or not. The opportunistic demands always have to be satisfied, the simulator affects the trip to a free taxi located at the same geographical point. As we can note, an opportunistic demand is only considered when it exists spatial and temporal coincidence between the demand and a free taxi, otherwise the demand is simply ignored. It is worth recalling that the opportunistic demands may lead to unsatisfied booking demands initially accepted by the taxi dispatching system.

5.2 Taxi dispatching system

A fleet management system aiming to overcome the weakness of the real-life rule-based taxi dispatching system is proposed in this section. The main objectives of the system are to maximize the number of accepted customers and to minimize the customer waiting time. One of the major issues is how to deal with opportunistic demand. Indeed, this kind of demand is unpredictable and must always be satisfied, so free taxis must be at any moment able to satisfy the longest trip without running out of charge. This constraint makes the problem considerably more complex forcing the system to provide a mechanism ensuring the feasibility of the already accepted trips each time an opportunistic demand is accepted.

The approach proposed consists in maintaining continuously a feasible planning for the taxis and the charging terminals (see Figure 1). Each time a customer asks for a trip, a simple *insertion algorithm* is run, at the end of which either the trip has been successfully inserted or not. The objective is to assign the customer to the taxi minimizing the customer waiting time (a parameterizable announced customer waiting time can be authorized). If none of the tried delays on the pick-up time leads to a feasible planning, a *rescheduling algorithm* allowing to reallocate the already accepted customers to the taxis is run.

In all these processes, a key routine which schedules the charging tasks of a taxi, given a planning for the other taxis and the charging terminals is often called. It consists in a greedy algorithm aiming to insert a charging task between each pair of successive trips of the same route.

In the case of an opportunistic demand, which is necessarily accepted, we follow exactly the same scheme except the fact that there is no degree of freedom in the insertion process: the trip is inserted at the front of the planning of the taxi stopped by the customer, and the rescheduling algorithm is also run if it is necessary.

In the next sections, the algorithms integrating the fleet management system are briefly described. Let us first introduce some notations. Let CR_i be a booking customer request. Each customer request CR_i is defined by a start time S_i and an origin-destination pair $O_i - D_i$. The S_i is fixed by the customer when the customer request arrives. The completion time of a trip is $C_i = S_i + \tau_{O_i D_i}$, where $\tau_{O_i D_i}$ is the travel time between the origin and destination of the customer request CR_i. Finally, let $R : CT_j$ be a taxi charging task scheduled on the charging terminal CT_j.

5.2.1 Insertion algorithm

This algorithm is the first step in order to decide if a new trip CR_{new} is accepted or not. The objective is to assign the trip to the taxi minimizing the delay on the pick-up time. The algorithm increasely tests the different authorized pick-up times. Once the start time is fixed, we sequentially try for each vehicle to insert the new request. First the scheduled charging tasks are removed. Then the new request is accepted only if it can be inserted with no constraint violation (the pick-up times of the rest of customers are respected and the current autonomy of the vehicle, without any charging task, is sufficient). In the case that the vehicle autonomy-related constraint is violated, a greedy algorithm trying to schedule a charging task between each pair of trips is proposed. After the charging tasks are inserted, if the taxi is able to perform the trips without running out of charge, then the customer request is also accepted.

5.2.2 Rescheduling algorithm

The rescheduling algorithm is proposed when the new customer is still not accepted after the insertion algorithm. As for the insertion algorithm, the goal is to find a new feasible planning for the vehicles integrating the new request CR_{new}. The main difference is that the trips can be reassigned to different vehicles.

The problem without taking into account the autonomy-related constraints can be solved in polynomial time (Neumann, Schwindt and Zimmermann, 2002). The idea is to convert the schedule of trips (without the charging tasks) into a graph and to verify using a max flow computation that all trips can be performed by the taxis. To construct the network two vertices are considered for each customer request CR_i, the first one represents the pick-up time v_i and the second one the completion time v'_i of the customer request. Four dummy vertices are required: 0, $0'$, a source s and a sink t. The arcs are $(s, 0)$, $(0', t)$, all the (s, v_i), all the (v'_i, t), all the $(0, v'_i)$, all the $(v_i, 0')$, and all the (v'_i, v_j) such that the customer request CR_j can be performed by the same taxi than the customer request CR_i and after CR_i, that means if $S_j \geq C_i + \tau_{D_i O_j}$. Except the arcs $(s, 0)$ and $(0', t)$, they all have a capacity equal to 1. The arcs $(s, 0)$ and $(0', t)$ have a capacity equal to n. A maximum flow is this directed graph determines the schedule feasibility and also proposes a new planning for the vehicles respecting the pick-up times of the customers.

The max flow computation is integrated in the rescheduling algorithm in order to check the feasibility of the schedule for a given pick-up time in $[S_{new}, S_{new} + \Delta]$ and, if it is the case, to find a reference planning (planning without charging tasks). A local search explores the neighborhood of the reference planning defined by the *swap* and the *reallocation* operators (Savelsbergh, 1992). Finally, for each explored planning respecting temporal constraints, the greedy algorithm for charging task scheduling is sequentially applied to the taxis that do not satisfy autonomy-related constraints (that is, taxis whose current charge is not enough to realize all the trips assigned to them without adding charging tasks). If a feasible solution is found, the new customer is then accepted.

6 COMPUTATIONAL EXPERIMENTS

The linear models for charging terminals location presented in Section 4 have been compared and evaluated by simulation on a set of randomly generated instances. Two set of instances have been generated considering different values for the average number of demands by unit of time. We consider then a first set of instances with a *weak demand* ($\lambda_{book}^{weak} \approx 0.4$ and $\lambda_{opp}^{weak} \approx 1.0$) and a second set of instances with a *strong demand* ($\lambda_{book}^{strong} \approx 0.8$ and $\lambda_{opp}^{strong} \approx 2.0$). Each set is composed of 60 instances (10 instances for each combination) generated from different values for the number of terminals ($r = \{5, 20, 40\}$) and for the number of vehicles ($n = \{100, 200\}$). The computation experiments consist on a 900 minutes simulation. The maximal authorized delay on pick-up time is fixed to $\Delta = 15$ minutes and the minimal charging time for a vehicle is fixed to 10 minutes.

Table 1 shows the results for the comparison between both models for the 40 instances with 5 charging terminals. The first column (*NbTrips*) displays the average of accepted customers (booking and opportunistic demands). *NbBooking* is the average of accepted booking requests. The average percentage of operating time and the average percentage of time when a taxi is waiting for an available charging terminal are

$r = 5$ *weak demand*		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
$n = 100$	*P-median model*	481.6	359.2	85.41 %	9.52 %
	Demand-based model (d_{out})	497.6	362.0	90.98 %	4.67 %
	Demand-based model (d_{in})	488.7	359.8	91.84 %	3.63 %
	Demand-based model (d_{mix})	488.5	358.9	91.17 %	4.41 %
$n = 200$	*P-median model*	551.6	366.9	90.16 %	7.06 %
	Demand-based model (d_{out})	567.9	368.3	94.97 %	2.80 %
	Demand-based model (d_{in})	563.3	365.8	94.99 %	2.70 %
	Demand-based model (d_{mix})	560.3	366.9	94.95 %	2.80 %
strong demand					
$n = 100$	*P-median model*	744.3	654.1	55.19 %	38.04 %
	Demand-based model (d_{out})	777.1	672.7	64.08 %	29.44 %
	Demand-based model (d_{in})	803.4	678.7	68.83 %	24.28 %
	Demand-based model (d_{mix})	789.9	674.3	65.63 %	27.67 %
$n = 200$	*P-median model*	886.0	703.9	66.01 %	30.33 %
	Demand-based model (d_{out})	943.5	725.0	75.04 %	21.48 %
	Demand-based model (d_{in})	943.0	718.5	77.30 %	19.09 %
	Demand-based model (d_{mix})	953.0	723.8	77.43 %	19.03 %

Table 1: Comparison between different programming models for instances with 5 charging terminals

$r = 20$ *weak demand*		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
$n = 100$	*P-median model*	497.0	361.4	94.46 %	0.19 %
	Demand-based model (d_{out})	495.9	361.4	93.99 %	0.28 %
	Demand-based model (d_{in})	499.0	362.7	94.19 %	0.22 %
	Demand-based model (d_{mix})	495.1	363.4	93.98 %	0.31 %
$n = 200$	*P-median model*	566.7	367.0	96.79 %	0.11 %
	Demand-based model (d_{out})	564.5	367.9	96.49 %	0.17 %
	Demand-based model (d_{in})	561.2	368.5	96.57 %	0.16 %
	Demand-based model (d_{mix})	561.1	368.5	96.49 %	0.20 %
strong demand					
$n = 100$	*P-median model*	894.8	733.7	90.02 %	0.89 %
	Demand-based model (d_{out})	895.4	734.5	88.97 %	1.26 %
	Demand-based model (d_{in})	884.7	733.4	89.21 %	1.15 %
	Demand-based model (d_{mix})	895.5	737.1	88.59 %	1.36 %
$n = 200$	*P-median model*	994.9	741.2	93.85 %	0.73 %
	Demand-based model (d_{out})	1005.4	744.8	93.40 %	0.88 %
	Demand-based model (d_{in})	997.0	741.1	93.28 %	0.87 %
	Demand-based model (d_{mix})	1008.9	744.0	93.19 %	0.95 %

Table 2: Comparison between different programming models for instances with 20 charging terminals

$r = 40$ *weak demand*		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
$n = 100$	*P-median model*	492.6	362.1	94.88 %	0.06 %
	Demand-based model (d_{out})	493.8	361.2	94.71 %	0.06 %
	Demand-based model (d_{in})	493.3	362.5	94.85 %	0.04 %
	Demand-based model (d_{mix})	492.8	361.8	94.69 %	0.05 %
$n = 200$	*P-median model*	549.7	366.9	97.08 %	0.03 %
	Demand-based model (d_{out})	556.1	366.1	97.04 %	0.03 %
	Demand-based model (d_{in})	560.5	366.6	97.06 %	0.03 %
	Demand-based model (d_{mix})	559.1	367.9	96.99 %	0.02 %
strong demand					
$n = 100$	*P-median model*	902.1	731.6	91.30 %	0.24 %
	Demand-based model (d_{out})	897.6	732.9	91.66 %	0.13 %
	Demand-based model (d_{in})	896.4	732.7	91.44 %	0.14 %
	Demand-based model (d_{mix})	897.0	732.8	91.31 %	0.13 %
$n = 200$	*P-median model*	999.0	741.1	95.00 %	0.14 %
	Demand-based model (d_{out})	1009.3	741.4	94.96 %	0.09 %
	Demand-based model (d_{in})	1008.8	741.0	94.90 %	0.11 %
	Demand-based model (d_{mix})	1006.5	741.9	94.91 %	0.12 %

Table 3: Comparison between different programming models for instances with 40 charging terminals

displayed on the last two columns.

The demand-based model is generally more efficient than the *P*-median model. More customers are satisfied, the operating time of taxis is higher and the time waiting for an available charging terminal is drastically reduced. In some cases, this last value is even divided by two. The different ways proposed to estimate the pointwise demand have been also compared. We observe that no strategy overcomes clearly the others.

This results reflects the advisability of considering demands in problem model. Nevertheless, the results are less conclusive for a large number of charging terminals as Table 2 and Table 3 show for 20 and 40 charging terminals, respectively. Although demand-based models are generally more efficient than the classical model, the reduction of waiting time is less important than for the instances with 5 charging terminals. Finally, we observe that demand-based models are better adapted to rush hours because the efficiency of these models increases directly with the average number of demands by unit of time.

7 CONCLUSION

In this paper, we study the electric charging terminal location problem. An upper bound for the number of customers that can be served by the system has been first computed. Then, different models have been proposed to solve the problem. The first model searches to locate the terminals in order to minimize the sum of the distances between the geographical points and the nearest charging terminal. The second model is a demand-based mixed integer linear programming model that considers a pointwise demand in order to maximize the demand covered by a close charging terminal. The originality of the approach lies in the way how initial dynamic demands are estimated as static pointwise demands. Both models have been tested and compared on a set of realistic instances randomly generated. The results show that the proposed demand-based model generally improves the model minimizing the sum of distances.

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